`r2_cmc`: Two-Terminal Nonlinear Resistor Model

Synopsis (Version: 1.0; Revision: 0.0; Date: 2005 Nov 12)

The `r2_cmc` model is a nonlinear 2-terminal resistor model. The model does not include parasitic capacitances. As an option, the model can include self-heating; this form of the model is named `r2_et_cmc`. The nonlinearity form is from Agere Systems [1], and effectively implements first and second order electric field coefficients of resistance. `r2_cmc` is well behaved and does not have the numerical problems that can arise in polynomial models. Although empirical, the form of the nonlinearity can model data reasonably well, especially for velocity saturation effects which are important in short resistors.

Usage

(NOTE: exact usage may be simulator dependent; e.g. whether the local temperature rise node for self-heating is made available or not, and whether the initial instance key-letter “`r`” is required.)

With model card:

```
r<instanceName> (<node1> <node2>) <modelName> <instanceParameters>
.model <modelName> r2_cmc <modelParameters>
```

Without model card:

```
r<name> (<node1> <node2>) r=<resistanceValue> [tc1=<tc1Value>] [tc2=<tc2Value>]
```

Examples:

```
r137 (n1 n2) rnpoly1 w=1u l=10u
.model rnpoly1 r2_cmc
+ rsh=100.0 xl=0.2u xw=-0.05u
+ p3=0.12 q3=1.63 p2=0.014 q2=3.79
r138 (n2 n3) r=110.0 tc1=1.0e-3
```

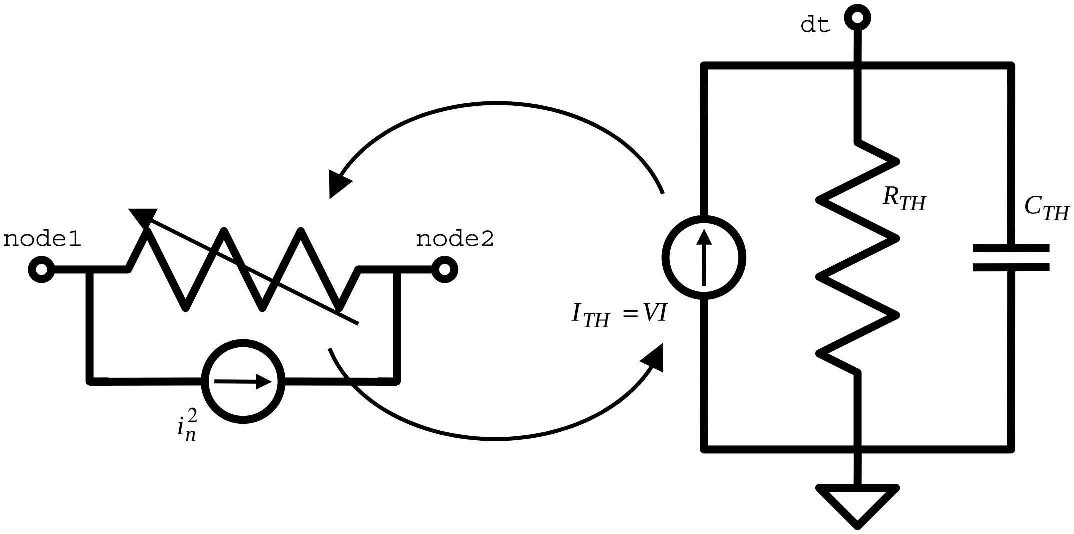

Fig. 1 `r2_cmc` Model Equivalent Network (the thermal sub-network is optional).

Parameters

NOTE: in this documentation parameters are set in `Courier New` font, to distinguish them from other quantities.

Name	Default	Min.	Max.	Units	Description
`m`	`1`	`0`	`inf`		multiplicity factor (number in parallel)
`w`	`1.0e-6`	`0.0`	`inf`	m	design width of resistor body
`l`	`1.0e-6`	`0.0`	`inf`	m	design length of resistor body
`r`	`100.0`	`0.0`	`inf`	Ω	resistance (per segment, total resistance is `r/m`)
`c1`	`1`	`0`	`1`		contact at terminal 1: 0=no 1=yes
`c2`	`1`	`0`	`1`		contact at terminal 2: 0=no 1=yes
`trise`	`0.0`			°C	local temperature delta to ambient (before self-heating)
`isnoisy`	`1`	`0`	`1`		switch for noise: 0=no 1=yes
`sw_et`	`1`	`0`	`1`		switch for turning off self-heating: 0=exclude 1=include

Table 1. Instance Parameters

Note that `c1`, `c2`, and `isnoisy` can be model parameters for the mode of use with a `.model` card. The instance parameter specification takes precedence over the `.model` card specification. The instance parameter `sw_et` is only valid for the electrothermal version of the model.

Name	Def	Min	Max	Units	Description
`version`					model version
`revision`					model revision (subversion)
`scale`	`1.0`	`0.0`	`1.0`		scale factor for instance geometries
`shrink`	`0.0`	`0.0`	`100.0`	%	shrink percentage for instance geometries
`tmin`	`-100.0`	`-250.0`	`27.0`	°C	minimum ambient temperature
`tmax`	`500.0`	`27.0`	`1000.0`	°C	maximum ambient temperature
`rthresh`	`1.0e-3`	`0.0`	`inf`	Ω	threshold to switch to resistance form

Table 2. Special Model Parameters (may link to global simulator parameters)

Name	Def	Min	Max	Units	Description
`level`	`1002`				model level
`tnom`	`27.0`	`-250.0`	`1000.0`	°C	nominal (reference) temperature
`rsh`	`100.0`	`0.0`	`inf`	Ω /☐	sheet resistance
`lmin`	`0.0`	`0.0`	`inf`	µm	minimum allowed drawn length
`lmax`	`9.9e99`	`0.0`	`inf`	µm	maximum allowed drawn length
`wmin`	`0.0`	`0.0`	`inf`	µm	minimum allowed drawn width
`wmax`	`9.9e99`	`0.0`	`inf`	µm	maximum allowed drawn width
`xw`	`0.0`			µm	width offset (total)

Name	Def	Min	Max	Units	Description
xl	0.0			µm	length offset (total)
dxle	0.0			µm	length delta for field calculation
sw_efgeo	0	0	1		switch for electric field geometry calculation: 0=design 1=effective
q3	0.0	0.0	inf	µm/V	1/field at which the linear field coefficient activates
p3	0.0	0.0	1.0		linear field coefficient factor: E_{C1}=p3*q3
q2	0.0	0.0	inf	µm/V	1/field at which the quadratic field coefficient activates
p2	0.0	0.0	1.0-p3		quadratic field coefficient factor: E_{C2}=p2*q2^2/2
kfn	0.0	0.0	inf		flicker noise coefficient (unit depends on afn)
afn	2.0	0.0	inf		flicker noise current exponent
bfn	1.0	0.0	inf		flicker noise 1/f exponent
sw_fngeo	0	0	1		switch for flicker noise geometry calculation: 0=design 1=effective
jmax	100.0	0.0	inf	A/um	maximum current density
tminclip	-100.0	-250.0	27.0	°C	clip minimum temperature
tmaxclip	500.0	27.0	1000.0	°C	clip maximum temperature
tc1	0.0			/K	resistance linear TC
tc2	0.0			$/K^2$	resistance quadratic TC
tc1l	0.0			µm /K	resistance linear TC length coefficient
tc2l	0.0			µm $/K^2$	resistance quadratic TC length coefficient
tc1w	0.0			µm /K	resistance linear TC width coefficient
tc2w	0.0			µm $/K^2$	resistance quadratic TC width coefficient
tc1kfn	0.0				flicker noise coefficient linear TC
gth0	1.0e+6	0.0	inf	W/K	thermal conductance fixed component
gthp	0.0	0.0	inf	W/Kµm	thermal conductance perimeter component
gtha	0.0	0.0	inf	W/Kµm^2	thermal conductance area component
cth0	0.0	0.0	inf	sW/K	thermal capacitance fixed component
cthp	0.0	0.0	inf	sW/Kµm	thermal capacitance perimeter component
ctha	0.0	0.0	inf	sW/Kµm^2	thermal capacitance area component

Table 3. Model Parameters

Note that `tc1` and `tc2` can be instance parameters for the direct instantiation mode of use, without a `.model` card. The thermal conductance and capacitance parameters are only valid for the electrothermal version of the model.

Bias Dependence

If V is the voltage between the terminals `node1` and `node2`, then the current flowing from `node1` to `node2` is

(1) $$I = \frac{V}{r\_dc},$$

the DC bias dependent resistance is

(2) $$r\_dc = R_0(T) \cdot rFactor$$

where $R_0(T)$ is the zero-bias resistance at the device temperature (which includes self-heating for the electrothermal version of the model), and the bias-dependent resistance factor is

(3) $$rFactor = 1 - \mathtt{p2} - \mathtt{p3} + \mathtt{p2}\sqrt{1 + (\mathtt{q2}E)^2} + \mathtt{p3}\sqrt[3]{1 + |\mathtt{q3}E|^3}\ .$$

The electric field E can be calculated based on either the design length or the effective electrical length, depending on the switch parameter `sw_efgeo`. This is because, depending upon how the model is used and the model parameters are characterized, there can be effects in the value of `xl` (the parameter that defines the difference between the design length and the effective electrical length) that are not related to velocity saturation (e.g. if end spreading and/or contact resistance are included in `xl`). If `sw_efgeo` is 1 (“true”) then the electric field is calculated from the effective geometry,

(4) $$E = \frac{V}{leff\_um + \mathtt{dxle}}$$

otherwise it is calculated from the design geometry,

(5) $$E = \frac{V}{l\_um + \mathtt{dxle}}\ .$$

In both cases, an additional length offset `dxle` is included, to allow flexibility and optimization in fitting data by separating the lengths used for resistance and field nonlinearity calculation. Although there is a singularity at $V = E = 0$ because of the absolute value operation in (3), the derivative of (1) with respect to voltage is defined and continuous up to third order. The fourth order derivative does not exist at $V = 0$ and its left limit does not equal its right limit there.

For the electrothermal version of the model, the thermal power is calculated as

(6) $$I_{TH} = V \cdot I$$

and the powers that flow through the thermal resistance and thermal capacitance are

(7) $$T(\mathtt{dt})g_{TH}$$

and

(8) $$ddt\left(T(\mathtt{dt})c_{TH}\right)$$

respectively, where $T(\mathtt{dt})$ is the local temperature rise due to self-heating and the thermal conductance and capacitance are g_{TH} and c_{TH}, respectively. The thermal admittance is $y_{TH} = g_{TH} + j\omega c_{TH}$.

Geometry Dependence

The basic calculation for the (zero bias) resistance of a resistor is $R_0 = \mathtt{rsh}\, L/W$. Because of several physical effects the length and width used in this calculation differ from the design (or mask) length and width that define the resistor layout. `r2_cmc` incorporates a simple, fixed offset between design and effective (electrical) length and width. Because subcircuit models for resistors can consist of multiple resistance sections connected in series, it is desirable to be able to switch on and off the “end corrections” for length to facilitate implementation of such multi-section models. This is the function of the `c1` and `c2` instance parameters of the `r2_cmc` model. The effective length offset is

(9) $$xleff = \mathtt{xl}(\mathtt{c1}+\mathtt{c2})/2$$

(which is zero if neither end is contacted, `xl` if both ends are contacted, and $\mathtt{xl}/2$ if only one end is contacted).

The design length and width, in units of microns, are

(10) $$l\_um = \mathtt{l} \cdot \mathtt{scale} \cdot (1-\mathtt{shrink}/100) \cdot 10^6,$$

(11) $$w\_um = \mathtt{w} \cdot \mathtt{scale} \cdot (1-\mathtt{shrink}/100) \cdot 10^6$$

where conversions from optical shrinking and unit scale conversion are included. If `scale` is 1 then `l` and `w` should be specified on model instances in meters, if `scale` is 1.0e-6 then `l` and `w` should be specified in units if microns. The effective electrical dimensions are

(12) $$leff\_um = l\_um + xleff,$$

(13) $$weff\_um = w\_um + \mathtt{xw}.$$

There are three modes of geometric calculation based on the instance parameters `w`, `l`, and `r`. All modes are based on resistance being specified (or calculated), at zero applied bias and at the nominal device temperature specified by the parameter `tnom`. The order of importance of considering the instance parameters is (in order of from most to least important) width, length, and resistance. If all are specified, the instance `r` value is ignored, and resistance is calculated from the specified length and width.

If length and resistance are specified, and width is not specified, then $R_{0,nom}$, the zero bias resistance at nominal temperature `tnom`, is

(14) $$R_{0,nom} = \mathtt{r},$$

and the effective width is calculated as

(15) $$weff\_um = \frac{\mathtt{rsh}}{R_{0,nom}} leff\_um$$

and possibilities of zero resistance or length, and error conditions of negative length or width, are handled.

If resistance is specified, and length is not specified, then

(16) $$R_{0,nom} = \mathtt{r},$$

(17) $$leff\_um = \frac{R_{0,nom}}{\mathtt{rsh}} weff\_um$$

and again possibilities of zero resistance and error conditions of negative length or width are handled.

For any other combination of instance parameter specification (resistance is not specified, or if it is then both width and length, which override resistance specification, are also specified), then the resistance is calculated from the geometry,

(18) $$R_{0,nom} = \mathtt{rsh}\frac{leff\_um}{weff\_um}$$

and zero resistance or conductance, and negative length or width errors, are handled.

Although end effects, such as spreading resistance and contact resistance, are assumed to be modeled via the `xl` parameter, the temperature coefficients of the end effects may differ from those of the body of the resistor. Simple analysis shows that these different temperature coefficients can be accounted for by introducing inverse length dependence to the temperature coefficients. A width dependence of temperature coefficients of resistance is also included in the model. Therefore in `r2_cmc`

(19) $$T_{C1}^{eff} = \mathtt{tc1} + \frac{0.5(\mathtt{c1}+\mathtt{c2})\mathtt{tc1l}}{leff\_um} + \frac{\mathtt{tc1w}}{weff\_um},$$

(20) $$T_{C2}^{eff} = \mathtt{tc2} + \frac{0.5(\mathtt{c1}+\mathtt{c2})\mathtt{tc2l}}{leff\_um} + \frac{\mathtt{tc2w}}{weff\_um}$$

where the length dependence is switched on, off, or halved, depending on whether the resistor is contacted at both ends, not contacted, or contacted at only one end, respectively. The dependence of the temperature coefficients on whether a resistor is contacted or not enables consistent modeling of temperature coefficients for single or multiple section models.

The thermal conductance and capacitance include area, perimeter, and fixed components. Asymptotically for a large area device, the heat flow is perpendicular to the plane of heat generation in the resistor, and the heat energy stored in a device depends on its volume, hence the area dependent component. For a long resistor, as it becomes narrower more of the heat flow is conducted by a “fringe” path at the edges of the device, hence the perimeter dependent component. As both length and width decrease, the thermal conditions in the device asymptotically approach that of a point source in an infinite medium, hence the fixed component. The thermal conductance and capacitance are therefore

(21) $$g_{TH} = \mathtt{gth0} + \mathtt{gthp} \cdot p\_um2 + \mathtt{gtha} \cdot a\_um2$$

(22) $$c_{TH} = \mathtt{cth0} + \mathtt{cthp} \cdot p\_um2 + \mathtt{ctha} \cdot a\_um2$$

where the area and perimeter are calculated as

(23) $$a\_um2 = l\_um \cdot w\_um$$

(24) $$p\_um = 2l\_um + (\mathtt{c1}+\mathtt{c2})w\_um\,.$$

The calculated perimeter therefore depends on whether the ends are contacted or not. Note that often the design dimensions of the body of a resistor differ from the overall dimensions of the device, for example if the design length is considered to be the unsalicided length of a poly resistor, the total resistor length will typically include silicided contact regions. So it is not readily apparent what dimension should be used in calculation of the thermal conductance and capacitance. That is why the design dimensions, rather than some effective dimensions (whose value is calculated to best fit DC electrical data), are used. This turns out to be fine (with the exception that differences between the perimeter components along length and width dimensions are ignored), because if there is some difference Δ between design and effective dimensions for thermal conductance modeling, then for a device contacted at both ends

$$
\begin{aligned}
g_{TH} &= \mathtt{gth0} + \mathtt{gthp}\left(2l\_um + 2w\_um + 4\Delta\right) + \mathtt{gtha}\left(l\_um + \Delta\right)\left(w\_um + \Delta\right) \\
&= \left(\mathtt{gth0} + 4\mathtt{gthp}\cdot\Delta + \mathtt{gtha}\cdot\Delta^2\right) + \left(\mathtt{gthp} + 0.5\mathtt{gtha}\cdot\Delta\right)p\_um + \mathtt{gtha}\cdot a\_\mathtt{um2}
\end{aligned}
\tag{25}
$$

therefore any difference between design and effective dimensions can be taken into account by appropriate characterization of the fixed, perimeter, and area component parameters.

Because the “local” thermal conductance differs between the edge of a device and the center of a device, it is higher at the edge because of “fringing” conductance, the temperature of a resistor undergoing self-heating is not spatially uniform, but is lower at the edges than in the middle. This is not taken into account in the `r2_cmc` model.

Temperature Dependence

The zero-bias resistance R_0 varies with temperature as

(26) $$R_0(T) = R_{0,nom}\left(1 + T_{C1}^{eff}\,dT + T_{C2}^{eff}\,dT^2\right)$$

where $R_{0,nom}$ is the nominal value of the zero-bias resistance, at the nominal temperature `tnom`, dT is the temperature difference (including self-heating, if that form of the model is used) with respect to `tnom`, and T_{C1}^{eff} and T_{C2}^{eff} are first (linear) and second (quadratic) order temperature coefficients. These coefficients have both a width dependence and a length dependence, the latter to enable modeling of resistors that have different temperature coefficients for “end” resistance (which includes contacts and contact enhancement regions) compared to “body” resistance without having to implement a sectional (subcircuit) model with explicit end and body resistance components. The width and length dependency is detailed in the section on geometry dependence; see (19) and (20). Smooth limiting of the resistance temperature coefficient in (26) is implemented to limit its lower value to 0.01.

For the isothermal version of the model the temperature difference dT is calculated statically based on the device temperature (which can vary from the circuit ambient temperature by setting the instance parameter `trise`, which is the local device temperature difference with respect to the circuit ambient temperature). For the electrothermal version of the model dT is calculated dynamically and self-consistently with the power dissipation of the device.

The flicker noise coefficient varies with temperature as

(27) $$K_{FN}(T) = \mathtt{kfn}\left(1 + \mathtt{tc1kfn}\,dT\right)$$

where `kfn` and `tc1kfn` are model parameters.

Noise

The noise model comprises two components, a thermal (white) noise component and a flicker ($1/f$) noise component. These components are noise current spectral density (in A^2/Hz) that are implemented as a noise current sources in parallel with the resistance element.

The thermal noise component is based on the DC conductance of the device,

$$(28) \qquad i_{thermal}^2 = 4kT_K G_0(T)/rFactor$$

where k is Boltzmann's constant, T_K is the device temperature (in Kelvin, including the effect of self-heating), G_0 is the zero-bias conductance of the resistor (at the temperature T), and $rFactor$ is the bias-dependent (DC) resistance factor (3).

The flicker noise component is DC current dependent [2], and scales with geometry per the physical restrictions noted in [3],

$$(29) \qquad i_{flicker}^2 = K_{FN}(T)\left(\frac{I}{W}\right)^{\mathtt{afn}} \frac{W}{L} \frac{1}{f^{\mathtt{bfn}}}$$

where f is frequency (in Hz), `afn` and `bfn` are model parameters, $K_{FN}(T)$ is the temperature dependent flicker noise coefficient (27), I is the DC current in the resistor, and W and L are the resistor width and length, respectively, in units of micron (µm). If the switch parameter for flicker noise geometry calculation `sw_fngeo` is 0 ("false") then W and L are design geometries, w_um and l_um respectively, else if it is 1 ("true") then W and L are effective geometries, $weff\_um$ and $leff\_um$ respectively.

Note that if self-heating is included, then possibly there is a frequency dependence to the flicker noise because of the thermal time constant. There is no data to verify this at present so a frequency independent noise current spectral density is used.

Operating Point Information

Name	Units	Description
`v`	V	voltage across resistor
`i`	A	current through resistor
`power`	W	dissipated power
`r0`	Ohm	zero-bias resistance (per segment)
`leff_um`	µm	effective electrical length in µm
`weff_um`	µm	effective electrical width in µm
`r_dc`	Ohm	DC resistance (including bias dependence and `m`)
`r_ac`	Ohm	AC resistance (including bias dependence and `m`)
`rth`	K/W	thermal resistance
`cth`	sW/K	thermal capacitance
`dt_et`	K	self-heating temperature rise

Table 4. Operating Point Parameters

Note that `r0` is the resistance per segment (which matches the instance parameter `r`), all other quantities include the effect of scaling by the multiplicity parameter `m`.

Description and Details

The voltage nonlinearity of the device resistance is [1]

$$R(E) = R_0\left(\left(1 - \mathtt{p2} - \mathtt{p3}\right) + \mathtt{p3}\sqrt[3]{1 + \left|\mathtt{q3}E\right|^3} + \mathtt{p2}\sqrt{1 + \left(\mathtt{q2}E\right)^2}\right) \tag{30}$$

where R_0 is the zero bias resistance of the resistor, $E = V/L$ is the electric field across the device, `p3` and `q3` are parameters of the effective first order (linear) electric field coefficient, and `p2` and `q2` are parameters of the effective second order (quadratic) electric field coefficient. Because the nonlinearity is based on field, rather than voltage, it scales with geometry.

For $\left|\mathtt{q3}E\right|$ somewhat greater than 1, the cubic component of the model becomes

$$R(E) = R_0\left(1 + \mathtt{p3}\left(\left|\mathtt{q3}E\right| - 1\right)\right) \tag{31}$$

therefore this term approximates a linear (first order) field dependence of resistance with a coefficient of value $\mathtt{p3} \cdot \mathtt{q3}$.

For $\left|\mathtt{q2}E\right|$ somewhat less than 1, the quadratic component of the model becomes

$$R(E) = R_0\left(1 + 0.5\mathtt{p2} \cdot \mathtt{q2}^2 E^2\right) \tag{32}$$

therefore this term approximates a quadratic (second order) field dependence of resistance with a coefficient of value $0.5\mathtt{p2} \cdot \mathtt{q2}^2$. For high fields this component becomes

$$R(E) = R_0\left(1 + \mathtt{p2}\left(\left|\mathtt{q2}E\right| - 1\right)\right) \tag{33}$$

and it turns out that the behavior embodied in (32) and (33) is quite accurate for modeling velocity saturation, which is a significant component of nonlinearity for shorter resistors.

To ensure that the resistance does not become negative,

$$0 \le \mathtt{p3} < 1 \tag{34}$$

and

$$0 \le \mathtt{p2} < 1 - \mathtt{p3} \tag{35}$$

are enforced. This also precludes the model exhibiting a negative differential resistance (NDR). NDR is observed in some devices, but this is from self-heating effects in resistors with positive temperature coefficients. This behavior should therefore be modeled using the electrothermal version of the model.

In SPICE-like simulators, which are based on modified nodal analysis (MNA), it is preferable to formulate models as voltage controlled current sources (VCCS's). This is the default for the `r2_cmc` model. For small resistance values this can cause numerical problems, and the MNA formulation is not possible for zero valued resistors (which have infinite conductance). For small resistance values it is better to switch to a current controlled voltage source (CCVS) formulation. Implicitly, this increases the matrix size for MNA analysis, as the current through the CCVS becomes a system variable. The `r2_cmc` model includes a parameter `rthresh`, and if the total (not per segment, but `r/m`) resistance at zero bias is less than `rthresh` then the model switches to a CCVS formulation, for numerical stability and to be able to work properly for zero valued resistors. Note that this makes the model implicit as the formulation is effectively $V = I \cdot R(V)$.

Notes on Parameter Extraction

This section provides some information that can help in setting up parameter extraction algorithms. It describes techniques to get initial values that can then be refined by optimization. It does not give a complete and perfect procedure for parameter extraction.

For characterization of `rsh`, `xl`, and `xw` use zero bias resistance data from (at least) three different device geometries. Usually these would be two devices of the same length and different widths, and another device with the same width as one of the other devices and a different length. Strictly from the point of view of simplifying parameter extraction, these would be a long/wide resistor, a long/minimumWidth resistor, and a minimumLength/wide resistor. However short/wide resistors can be difficult to measure, especially for low sheet resistance devices, because the low resistance leads to high currents and compliance limits on test equipment, and because data from low resistance devices is more liable to difficulties of interpretation because of sensitivity to parasitic metal interconnect, probe to pad, and cabling resistances. (For low resistance devices use Kelvin measurements to ensure that data are not corrupted by these parasitics.) Also, for characterization it is desirable, where possible, to use geometries that are representative of those used in real circuits, because the goal of modeling is not to get the “best possible” (i.e. “most physical”) parameter values but to provide an overall model that most accurately describes the electrical behavior of devices for geometries and biases that are most relevant for circuit design. Minimum width resistors maximize the observability of `xw` in data, and so should be used for characterization test structures. If there is a preferred (“recommended design minimum”) width that, to be less sensitive to variations and processing and device limits, is not too much greater than the true technology minimum but expected to be widely used in circuits, then that width could be used instead of the minimum. High value resistors are commonly long and narrow, to minimize area, and also allow `xw` characterization relatively independently of `xl` and webbing (dog-boning) effects, therefore a long/narrow (where narrow is either minimumWidth or near this) and a long/wide (where wide would be roughly 5 times or more the minimum width) device should form part of the characterization set. A short/narrow or short/wide device should then also be included. The wide device reduces sensitivity of `xl` characterization to `xw`, and can be important in circuits (e.g. in some ESD protection circuits). If this generates too small a resistance to easily measure, or it is expected that short/narrow resistors are more important in circuits for target application areas, then a short/narrow device should be used.

For 3 (or more) separate geometries, measure the zero bias resistance at the nominal temperature. This can be done by extrapolation from measured $R(V)$ data to zero bias, or from an AC conductance measurement. Be careful when extrapolating using resistance (or conductance) calculated from $I(V)$ data as this tends to get noisy for low voltages. The reason for using the zero bias resistance is it is not affected by nonlinearities or self-heating. If the geometries are L_i and W_i , these are design geometries in units of microns (10) and (11), and the zero bias resistances are $R_{0,i}$, for $i = 1,2,3,...$, then from the basic resistance form $R_{0,i} = \mathtt{rsh}(L_i + \mathtt{xl})/(W_i + \mathtt{xw})$

$$(36)\qquad \begin{bmatrix} 1 & L_1 & -R_{0,1} \\ 1 & L_2 & -R_{0,2} \\ 1 & L_3 & -R_{0,3} \end{bmatrix} \begin{bmatrix} \mathtt{rsh}\cdot\mathtt{xl} \\ \mathtt{rsh} \\ \mathtt{xw} \end{bmatrix} = \begin{bmatrix} R_{0,1}W_1 \\ R_{0,2}W_2 \\ R_{0,3}W_3 \end{bmatrix}.$$

This is a set of simultaneous equations that can be solved for $\mathtt{rsh}\cdot\mathtt{xl}$, `rsh` ,and `xw`, and thence `xl`. If data from more than 3 geometries are available then the equations are over-determined and can be solved using standard techniques (the Moore-Penrose pseudoinverse), which is a least squares solution for the parameters. The geometry selection for characterization should be such that the coefficient matrix in (36) is numerically well conditioned. If two devices of the same length but different widths are available then directly

$$(37)\qquad \mathtt{xw} = \frac{R_{0,1}W_1 - R_{0,2}W_2}{R_{0,2} - R_{0,1}} = \frac{G_{0,1}W_2 - G_{0,2}W_1}{G_{0,2} - G_{0,1}}$$

where $G_{0,i} = 1/R_{0,i}$. Similarly if two devices of the same width but different lengths are available then

(38) $$\mathtt{xl}=\frac{R_{0,1}L_2-R_{0,2}L_1}{R_{0,2}-R_{0,1}}=\frac{G_{0,1}L_1-G_{0,2}L_2}{G_{0,2}-G_{0,1}}\,.$$

If the zero bias resistance of a device is measured at two temperatures T_L and T_H , which would typically be the extremes (low and high) for the range the model was required to work over, as well as at `tnom`, then from (26)

(39) $$\begin{bmatrix} dT_L & dT_L^2 \\ dT_H & dT_H^2 \end{bmatrix}\begin{bmatrix} T_{C1} \\ T_{C2} \end{bmatrix}=\begin{bmatrix} \left(R_0(T_L)/R_{0,nom}\right)-1 \\ \left(R_0(T_U)/R_{0,nom}\right)-1 \end{bmatrix}$$

where $dT_L = T_L - \mathtt{tnom}$, $dT_H = T_H - \mathtt{tnom}$, and $R_{0,nom}$ is the zero bias resistance at the nominal temperature. This allows the temperature coefficients T_{C1} and T_{C2} to be calculated for this specified device.

Doing this for three different geometries (for simplicity of extraction these are wide/long, narrow/long, and wide/short resistors, however any three combinations that show significantly different temperature coefficients are acceptable, and as always if there are one or more specific geometries that are considered most important for design then they should be used) allows the first order temperature coefficients to be calculated from

(40) $$\begin{bmatrix} 1 & 1/\left(L_1+\mathtt{xl}\right) & 1/\left(W_1+\mathtt{xw}\right) \\ 1 & 1/\left(L_2+\mathtt{xl}\right) & 1/\left(W_2+\mathtt{xw}\right) \\ 1 & 1/\left(L_3+\mathtt{xl}\right) & 1/\left(W_3+\mathtt{xw}\right) \end{bmatrix}\begin{bmatrix} \mathtt{tc1} \\ \mathtt{tc1l} \\ \mathtt{tc1w} \end{bmatrix}=\begin{bmatrix} T_{C1}\left(L_1,W_1\right) \\ T_{C1}\left(L_2,W_2\right) \\ T_{C1}\left(L_3,W_3\right) \end{bmatrix}.$$

If only devices of different length are available for characterization, L_S and L_L (in design dimensions in microns), then

(41) $$\begin{bmatrix} 1 & 1/\left(L_S+\mathtt{xl}\right) \\ 1 & 1/\left(L_L+\mathtt{xl}\right) \end{bmatrix}\begin{bmatrix} \mathtt{tc1} \\ \mathtt{tc1l} \end{bmatrix}=\begin{bmatrix} T_{C1}(L_S) \\ T_{C1}(L_L) \end{bmatrix}.$$

A similar reduction of (40) can be used if only devices of different width are available. Analogous calculations follow for the second order temperature coefficient parameters `tc2`, `tc2l`, and `tc2w`.

Care needs to be taken in extraction of the parameters that control the bias dependence of the model; this is because nonlinearities from self-heating can be confounded with nonlinearities from other effects, especially velocity saturation. Consider a simple linear resistor with a first order temperature coefficient T_{C1} and a zero bias resistance $R_0 = \mathtt{rsh}L/W$, under DC biasing

(42) $$\frac{R}{R_0}=1+T_{C1}dT=1+T_{C1}R_{TH}IV\approx 1+T_{C1}R_{TH}\frac{V^2}{R_0}=1+\frac{T_{C1}}{\mathtt{rsh}\cdot\mathtt{gtha}}E^2\,.$$

where only the area component of the thermal conductance (21) has been included. Note that this has the same form of field dependence as the quadratic component (32), and so it can be difficult to distinguish what is the root cause of the nonlinearity, and hence which set of parameters to characterize, from $I(V)$ alone.

For isothermal modeling, where only the voltage coefficient parameters need to be characterized, two techniques can be used to generate initial estimates for these parameters, which should then be refined by optimization. The first technique is from [1]. From $R(E)$ for one device, which can be derived from DC $I(V)$ data (using design or effective length, whatever is preferred), calculate the first and second order field coefficients E_{C1} and E_{C2} from

(43) $$\frac{R}{R_0}=1+E_{C1}E+E_{C2}E^2\,.$$

This can be done by selecting two bias values E_1 and E_2, with measured DC resistances R_1 and R_2,and solving

$$(44)\qquad \begin{bmatrix} E_1 & E_1^2 \\ E_2 & E_2^2 \end{bmatrix}\begin{bmatrix} E_{C1} \\ E_{C2} \end{bmatrix}=\begin{bmatrix} (R_1/R_0)-1 \\ (R_2/R_0)-1 \end{bmatrix},$$

or by including data from more bias points and solving (44) in a least squares sense (i.e. via the Moore-Penrose pseudoinverse). Because $|\mathtt{q3}E|$ should be greater than 1 for the $(\mathtt{p3},\mathtt{q3})$ part of the model to be linear, `q3` can be approximated as the inverse of the minimum measured E, then comparing (31) and (43),

$$(45)\qquad \mathtt{p3}=E_{C1}/\mathtt{q3}\,.$$

Because $|\mathtt{q2}E|$ should be less than 1 for the $(\mathtt{p2},\mathtt{q2})$ part of the model to be quadratic, `q2` can be approximated as the inverse of the maximum measured E, then again comparing (31) and (43),

$$(46)\qquad \mathtt{p2}=2E_{C2}\big/\mathtt{q2}^2$$

gives a good initial estimate of `p2`.

Another approach to getting initial estimates for the electric field coefficient is as follows. For large $|\mathtt{q3}E|$ and $|\mathtt{q2}E|$ both (31) and (33) have the following form (for positive E)

$$(47)\qquad \frac{R}{R_0}=1+\left(\frac{E-E_{co}}{E_{cr}}\right)$$

where E_{co} is a corner field at which the resistance increase starts to occur and E_{cr} is the reciprocal of the slope of the increase of R/R_0 with field (if the root cause of the nonlinearity is velocity saturation E_{cr} turns out to be the critical field for velocity saturation). Differentiating R/R_0 with respect to E and finding the maximum value gives (the reciprocal of) E_{cr}, and then substituting the measured value of R/R_0 at the highest available E into (47) allows E_{co} to be calculated. Comparison of (47) with (31) and (33) gives

$$(48)\qquad \mathtt{q2}=\mathtt{q3}=1/E_{co}\,,\ \mathtt{p2}=\mathtt{p3}=0.5E_{co}/E_{cr}\ .$$

If both `p2` and `p3` are set to E_{co}/E_{cr} the nonlinearity is accounted for twice. For the goal of getting initial parameter estimates prior to optimization the nonlinearity is, without additional information, taken to consistent of similar linear and quadratic components, hence the factor of $1/2$ that is used in (48)

With initial estimates of the electric field coefficients from either of the above techniques, nonlinear least squares optimization can be used to refine the estimate. To improve fitting over geometry, the length offset for field calculated `dxle` can be included in the optimization, with an initial value of zero.

Note that parameter ranges restrict the model so that it can only be used for devices with positive electric field coefficients. Some devices exhibit apparent negative field coefficients at low bias, so that the resistance actually decreases with increasing bias. This is usually for devices (polysilicon resistors) with negative temperature coefficients, and the resistance decrease is from self heating. As the bias continues to increase (for short devices), the resistance is then observed to increase, because of velocity saturation. This change in behavior can be used to separate the data into regions where self heating is dominant and where velocity saturation (the electric field nonlinearity) is dominant. Initial estimates for the thermal resistance and bias nonlinearity parameters can be made from data in these regions, and refined by optimization.

If the self-heating model is being used, both thermal conductance and thermal capacitance parameters need to be characterized. This requires AC data. Also, although for devices with an apparent negative field coefficient at low fields it is possible to separate the effects of bias and self-heating nonlinearities in DC $I(V)$ data, this is not possible for devices with positive electric field coefficients at all biases. For such devices AC data is also required to allow characterization of the electric field dependence and thermal resistance parameters. The technique of [4] can be simplified to apply to resistors. The current in a resistor is

(49) $$I = I(V,T) = G(V,T) \cdot V$$

where G is the bias and temperature dependent DC conductance. Conservation of heat flow (the thermal through variable) at the local temperature node in the equivalent network for the model gives

(50) $$y_{TH}\, dT = I \cdot V \, .$$

If the AC current, voltage, and temperature are denoted $\tilde{i}$, $\tilde{v}$, and $\tilde{t}$, respectively, then

(51) $$\tilde{i} = \frac{\partial I}{\partial V}\tilde{v} + \frac{\partial I}{\partial T}\tilde{t} = \left(G + \frac{\partial G}{\partial V} V \right)\tilde{v} + g_t \tilde{t} = g\tilde{v} + g_t \tilde{t}$$

where g is the isothermal AC conductance and $g_t = \partial I / \partial T$ is the thermal transconductance, and

(52) $$y_{TH}\tilde{t} = (I + gV)\tilde{v} + g_t V \tilde{t} \, .$$

Solving for $\tilde{t}$ from (52) and substituting into (51) gives the total AC admittance for the resistor (including self-heating)

(53) $$y_{in} = g + \frac{g_t (I + gV)}{y_{TH} - g_t V}$$

and the thermal admittance is

(54) $$y_{TH} = g_t V + \frac{g_t (I + gV)}{y_{in} - g} \, .$$

As in [4], the thermal admittance can be calculated from (54) because I and V come from direct measurement, g_t can be calculated from the variation of I with temperature at a fixed bias, and from (53) g is the value of y_{in} at high frequencies. The thermal conductance can be calculated from the difference in low frequency and high frequency input admittance,

(55) $$g_{TH} = g_t V + \frac{g_t (I + gV)}{y_{in,LF} - g} = g_t V + \frac{g_t (I + gV)}{y_{in,LF} - y_{in,HF}} \, .$$

The thermal capacitance can be determined from the frequency dependence of y_{TH} .

A practical difficulty with this approach is that it is hard to get accurate data at frequencies sufficiently below the thermal pole. This is generally around 1-10MHz, so data should be available to about two orders of magnitude below this to allow proper determination of $y_{in,LF}$, yet s-parameter test sets these days emphasize high frequencies and it is difficult to get data below a few hundred kHz.

More information on self-heating characterization will be forthcoming.

Characterization of the thermal resistance and capacitance over geometry enables the constant, perimeter, and area components of thermal conductance and capacitance to be calculated. If any parameter P is a function of area A and perimeter P , then if its value is know for three different layouts, then

$$(56) \qquad \begin{bmatrix} 1 & P_1 & A_1 \\ 1 & P_2 & A_2 \\ 1 & P_3 & A_3 \end{bmatrix} \begin{bmatrix} P_C \\ P_P \\ P_A \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

allows the fixed, perimeter, and area components (P_C , P_P , and P_A) to be calculated.

If a device is affected by self-heating, the trend over geometry should be apparent. Specifically, for resistors of constant length, the effect of self-heating is seen to reduce as a resistor becomes narrower. This is because the “fringing” thermal conductance path has a proportionately greater effect for narrower resistors. The self-heating effect also reduces as length decreases.

For the $1/f$ noise parameters, measure the current noise for two or more bias values, chosen so that the $1/f$ noise is easily discernable from the white noise. Noise data are usually noisy, so filter to reduce the noise or for each bias fit a straight line to the noise current versus frequency data (on a log-log scale) for the portion of the data where $1/f$ noise dominates. If your data have spikes at the mains power supply frequency and its harmonics filter this out (or get a better test system). From (29)

$$(57) \qquad \ln\left(i_{flicker}^2\right) = \ln(\mathtt{kfn}) + \mathtt{afn}\ln(I/W) + \ln(W/L) - \mathtt{bfn}\ln(f)$$

so the (negative of the) slope of $\ln\left(i_n^2\right)$ versus $\ln(f)$ gives `bfn`. Select whether you want to scale noise with design or effective dimensions, then calculate I/W for the biases used for $1/f$ noise characterization (if you do not have a preference for either, characterize for both design and effective dimensions and see which approach best matches your data). For two or more biases regress $\ln\left(i_{flicker}^2\right)$ on $\ln(I/W)$, for a selected frequency, and the slope gives `afn`. `kfn` can then be directly calculated from any point.

Because noise data generally come from a complex measurement set up, that may include biasing resistors and potentiometers, filtering capacitors, and a low noise amplifier, it is possible for the data to have contributions and characteristics derived from these other components. The initially extracted noise parameters should therefore be refined based on simulations that as exactly possible mimic the whole test system setup; the parameters are optimized so the simulations best match the measured data (possibly filtered and definitely with supply frequency spikes removed).

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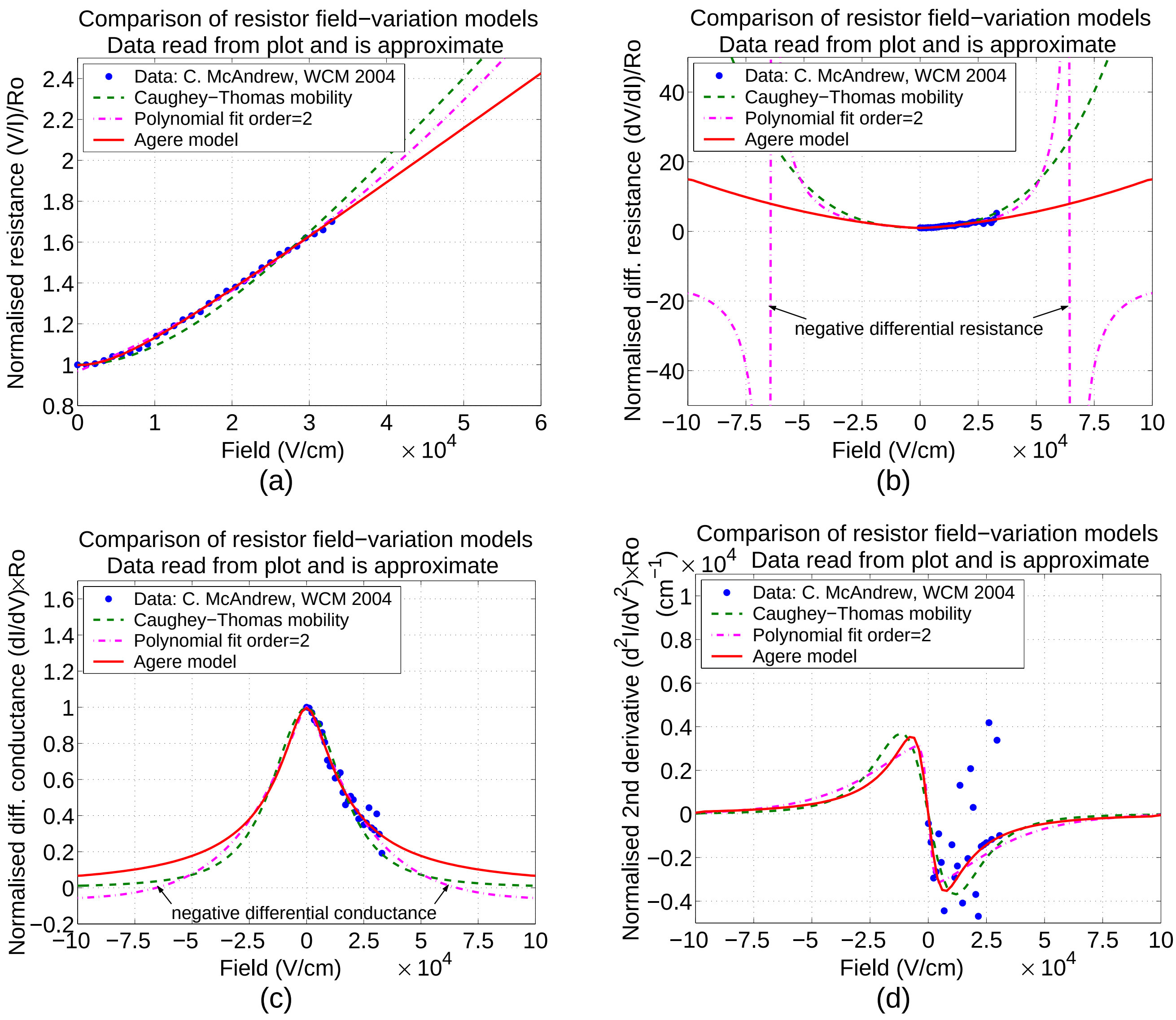
Modelling resistor voltage coefficients

Agere Systems Notes, June 2004

1. Issues with polynomial voltage coefficients in resistor models

Issues with polynomial voltage coefficients are: possible discontinuity at dR/dV at 0, zeros during Newton iteration and possible negative resistance. The effect of self-heating appears as a v^2 term in the measurements which cannot be separated from any other voltage effect. High-order voltage terms cause negative differential resistance (which is okay as long as they don't create negative resistance).



(a) (b) (c) (d)

Figure 1: Plots versus field of (a) normalised resistance, (b) normalised differential resistance, (c) normalised differential conductance and (d) second-derivative of current for measured data, Caughey-Thomas mobility model, polynomial fit and Agere model.

2. Agere resistor voltage coefficients: models and extraction

- Agere resistor voltage-coefficients

$$\text{R} = \text{Ro} \times \left[\text{(1-p2-p3) + p2 (1+(q2 E)}^{2}\text{)}^{1/2} + \text{ p3 (1+}|\text{q3 E}|^{3}\text{)}^{1/3} \right] . \qquad (1)$$

  Note: field (E) is used in the non-linear equation instead of voltage (V) otherwise the model is not scalable. Scalability requires that a resistor of length L1+L2 is equal to sum of resistor of length L1 and resistor of length L2. Using only the voltage in the non-linear equation will *not* satisfy scalability. Using field is both physically correct and scalable, because it is calculated as field = voltage/length of resistor.

- Polynomial coefficient model from fitting to data

  Using a polynomial coefficient model in terms of field:

$$\text{R} = \text{Ro} \times \left[\text{1 + c1}|\text{E}|\text{ + (c2 E)}^{2} \right] . \qquad (2)$$

  The polynomial coefficient model has coefficients obtained from fitting to the data. Coefficients from data shown in Fig. (1) are:

$$\begin{aligned} \text{c1} &= 1.5324 \times 10^{-5}\ \text{cm/V} \\ \text{c2} &= 1.4990 \times 10^{-5}\ \text{cm/V} \end{aligned} \qquad (3)$$

- Conversion from a polynomial coefficient model to Agere model

  The conversion from a polynomial coefficient model to Agere model in Eq. (1) is:

$$\begin{aligned} \text{q3} &\approx \text{1/min(measured E)} = 1 \times 10^{-3}\ \text{cm/V, a reasonable value} \\ \text{q2} &\approx \text{1/max(measured E)} = 2.5 \times 10^{-5}\ \text{cm/V, for data shown here} \\ \text{p3} &= \text{c1/q3} = 1.5324 \times 10^{-2}\ \text{(must always be} < 1) \\ \text{p2} &= 2 \times \text{c2}^{2}/\text{q2}^{2} = 7.1905 \times 10^{-1}\ \text{(must always be} < 1) \end{aligned} \qquad (4)$$

- Direct extraction of coefficients of Agere model is done by using a robust non-linear fitting software.

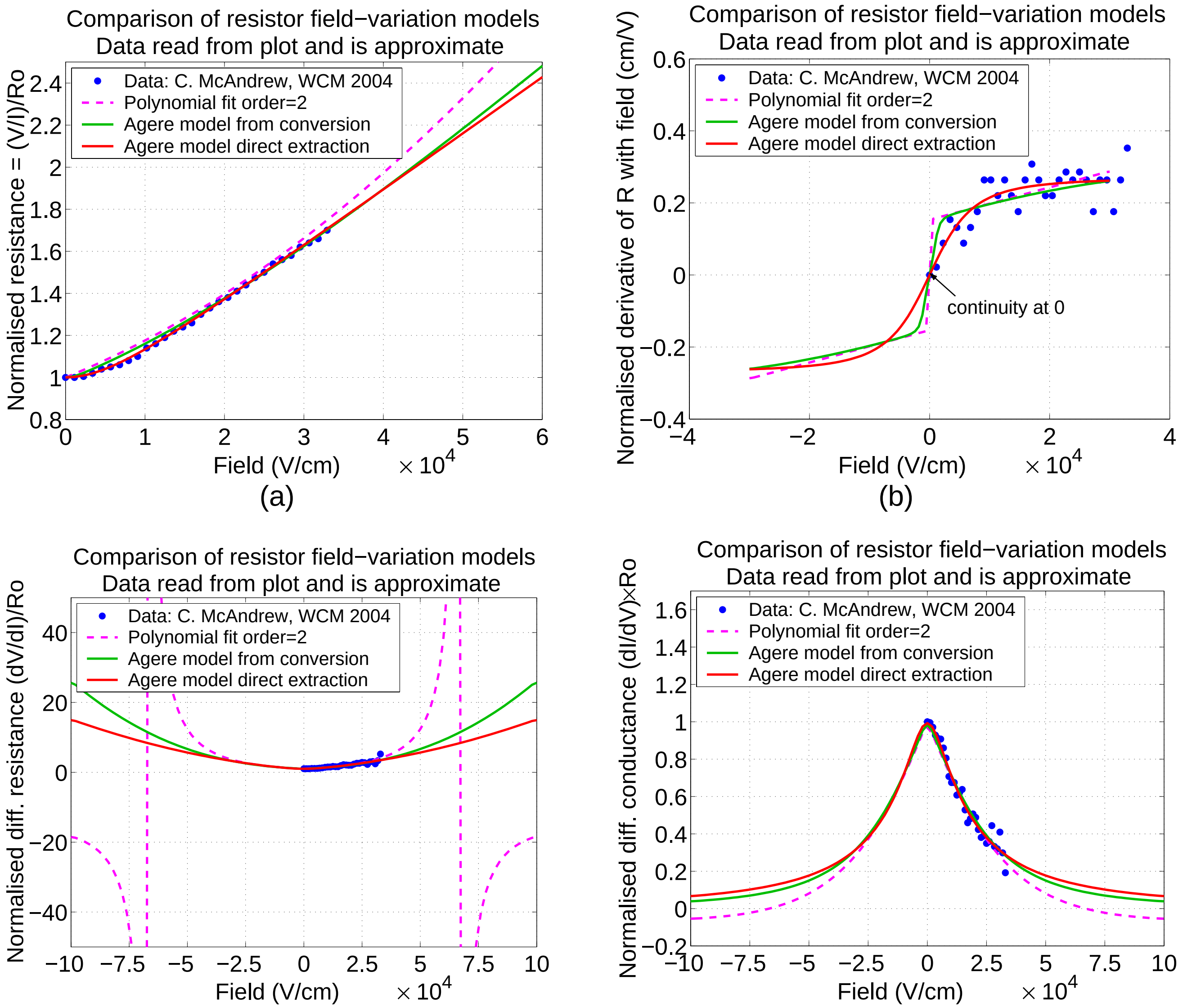
$$\begin{aligned} \text{q3} &= 1 \times 10^{-13}\ \text{cm/V}, \quad \text{q2} = 1.3111 \times 10^{-4}\ \text{cm/V}, \\ \text{p3} &= 7.9164 \times 10^{-1}, \quad \text{p2} = 2.0607 \times 10^{-1}. \end{aligned} \qquad (5)$$

3. **Note:** The coefficients above are chosen to be similar to the Caughey-Thomas mobility model [1]:

$$\mu(\text{E}) = \frac{\mu_0}{\sqrt{\text{1+(E/Ecrit)}^2}}, \quad \text{such that R} = \frac{1}{\text{q}\mu\text{(n+p)}} = \frac{1}{\text{q}\mu_0\text{(n+p)}}\sqrt{\text{1+(E/Ecrit)}^2}. \qquad (6)$$

Additional terms are added to the resistance for a better fit. This is similar to addition of mobility using Mathiessen's rule [2]. An important feature of this model is that in the limit $\text{E} \to \infty$, the resistance becomes linear with E (it can never go to zero or become negative in a simulator). However, this model also does not provide for negative differential resistance due to self-heating. Self-heating can be better handled by adding an electro-thermal network to the resistor model, similar to VBIC [3].

4. Comparison of polynomial coefficient and Agere models



(a) (b)

(c) (d)

Figure 2: Plots versus field of (a) normalised resistance, (b) normalised derivative of resistance with field, (c) normalised differential resistance and (d) normalised differential conductance for measured data, polynomial fit, Agere model from conversion of polynomial fit and Agere model from direct extraction.

5. References

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